



# Administration Guide to Advanced Preference Expression Methods

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## Preference Expression Facility Overview

PARTECS™ provides a powerful and easy-to-use Preference Expression facility to support the decision making process. In order to provide maximum flexibility, Administrators can configure the system and enable it to use several different algorithms, which implement different voting methods.

Preference Expression is the term used on PARTECS™ web application User Interface and refers to the entire engine of configuration/input/output of data of all available methods. For the technical purpose of this document, we'll adopt the term "voting", to stick with a simplest way of understanding their functionality and end purpose.

This is a list of the currently available voting methods, from the simplest to the most sophisticated state-of-the-art algorithms:

- Preferential Voting
- Approval Voting
- Borda Voting
- Instant Runoff Voting
- Condorcet Voting

It is extremely easy to implement variations of these voting methods; it is also easy to implement entirely different methods, if the customer requires it.

Common sophistications such as a quorum requirement (the election is valid only if a large enough percentage of the potential voters actually vote) and a supermajority requirement (the winner option must be vote by a large enough percentage of voters) are built-in into the PARTECS™ software.

In the following, we will informally discuss the pros and cons of the different voting methods. A formal technical discussion (for experts) is reported at the end (courtesy of <http://www.electionmethods.org/>).

## *Single Preference Voting (a.k.a. Plurality Voting)*

This is the voting method everybody is familiar with: the voter just selects the option he/she likes the most. The final result is computed according to the number of votes received for each candidate and the results are displayed in terms of percentages.

The single preference voting method is clear, simple and easy to grasp: it is the best method when you need to decide between two candidates. However, in the case of decisions involving three or more candidates, it is unfair with respect to minority candidates. The problem is that votes given to minority candidates are “wasted”, so people tend to vote the “lesser of two evils” between the first two major candidates. In the case of political elections, the usage of single-preference voting naturally leads to a two-party system (Duverger's Law) i.e. a duopoly regime. In the case of a decision making process this is less of an issue, but still one has the same undesirable effect: when there are two options, which are clearly favourites it becomes counter-productive to vote minority options and people are encouraged to vote insincerely.

Single-preference voting severely restricts the voter's expressiveness. Since the voter is forced to select just one candidate between many, he/she has no way to express the fact that he/she equally likes two or more candidates. Also, he/she cannot express the order of his/her preferences. If you have many options to choose from and want to give more freedom to your electors, it is best if you switch to a multiple choice voting method.

## *Approval voting*

Approval is the simplest multiple preference voting method. The voter just lists the options he/she likes in any order. At the end, the number of preferences for each candidate is counted and displayed in terms of percentages.

Approval method is easy to explain and to grasp, but restricts the voter expressiveness in the sense that he/she has no way to express that he/she likes a candidate more than another, since all listed candidates are considered equally liked. If you have many options to choose from and want to give more freedom to your electors, it is best if you switch to an ordinal voting method.

## *Borda voting*

Borda is the simplest ordinal voting method. It is also a rating method, since each candidate gets rated with a numbered ranking.

The elector lists the options he/she likes in a specific order, and each option gets a score according to its position. Options listed first get a higher score, so you may specify that you like candidate X more than Y just by listing Y after X.

Suppose for instance there are four options  $n=4$ : then the first option in the list gets  $n-1 = 3$  points, the second options  $n-2 = 2$  points, the third option  $n-3 = 1$  point and the last one gets no points at all. So, listing "B A C D" means:

```
B gets 3 points
A gets 2 points
C gets 1 point
D gets 0 points
```

Typically, in Borda method, electors are asked to list all the candidates in order to lower (but not to erase) the risk of abuses from insincere voters (see next section).

At the end, the total score for each candidate is counted and displayed as a percentage. The Borda method is theoretically very attractive, since it is relatively easy to understand and gives great expressive power to the user (effectively allowing him/her the possibility to rate the candidates in order of preference).

However, in practice, the Borda method is a rather poor method since insincere voters can easily abuse it. A simple example that shows why this is the case is reported below. If you want to use an ordinal method, which is solid against insincere votes, you should look at the Condorcet method.

### **Shortcomings of the Borda method**

Consider an election with four candidates A, B, C and D and the sincere electoral ballot.

```
3 : B A C D
2 : A C B D
```

(Three votes for "B A C D" and two votes for "A C B D"). The result is:

A: 12 points  
B: 11 points  
C: 7 points  
D: 0 points

In this case A wins over B since the last two voters, which preferred B to A, nevertheless liked A enough to put it as second choice. So, being sincere made their candidate lose the election. If B voters knew that their least preferred candidate D didn't have any hope to win, they could have listed it over A:

1: B A C D  
2: A C B D  
2: B D C A

Now the result of the election would have been:

B: 11  
A: 8  
C: 7  
D: 4

Strategically voting the weak candidate D over A made the B candidate win. Stealing points from a strong candidate and transferring them over a weak candidate is a good strategy to make middle candidates to win. So being sincere is not encouraged by the Borda method. In other words, the Borda method is not robust against insincere voters.

## *Instant Runoff (a.k.a. Preferential voting)*

Instant Runoff is an ordinal voting method, which may be a good choice for people familiar with runoff voting, i.e. voting in multiple turns, where weaker candidates are removed and their votes are transferred to the remaining candidates. It is simpler and less robust than Condorcet and more complicated but more robust than the Borda method. It is used in the real world in Australian elections under the name of Preferential voting (or Alternative voting).

Voters rank the candidates as first, second, third, etc. Table 1 illustrates the IRV tallying procedure for an example with four candidates (A, B, C, D) and 16 voters; see here for a more concrete example).

Table 1: An example ballot with four candidates A, B, C, D

3 : B A C  
2 : B C A  
1 : B D  
2 : A B D  
1 : A B C  
1 : A  
1 : D C B  
1 : D C A  
1 : D  
1 : C A B  
1 : C D A

The first step is to count the first choices. Candidate B got 6 of the 16 first choices, while A got 5, D got 3, and C got 2. If one candidate had received a majority of first choices, that candidate would have won, but nobody did in this case. The counting procedure therefore goes to a second round.

The candidate with the fewest first choices, candidate C, is now eliminated. Each vote for C is transferred to the next candidate. Thus, all C entries are eliminated and the remaining choices are pushed left, if necessary, to fill in the empty cells. Table 2 shows the result (the third column is no longer needed).

Table 2: Ballot after the first round

5 : B A  
1 : B D  
4 : A B  
1 : A D  
1 : A  
1 : D B  
2 : D A  
1 : D

The top choices are now counted again. Candidate A gained one new top choice (the second from last vote) for a total of 6. Still no candidate has a majority, so the counting procedure goes to a third round. Candidate D now has the fewest top choices and is therefore eliminated. Table 3 shows the result.

Table 3: Ballot after the first round

7 : B  
8 : A

In the final round, the third from last vote has been exhausted; so only 15 active votes remain. Candidate A picked up two votes and now has 8 votes, which is a majority of the remaining

votes, so candidate A wins. In this example, candidate A had fewer first choices than candidate B in the first round, but ultimately won the election.

The maximum number of rounds is always one less than the number of candidates. In the case of ties for the fewest top choices, the tied candidate with the fewest second choices is eliminated (if those are also tied, look at third choices, etc.). In the case of a tie in the final round, a coin toss can break the tie.

IRV has serious problems. It allows a sufficiently small minority of voters to safely register “protest” votes for minor-party candidates - but only as long as their candidate is sure to lose. As soon as their candidate threatens to actually win, they risk hurting their own cause by ranking their favourite first, just as they do under our current plurality system. IRV is therefore unlikely to be any more successful than plurality at solving the classic “lesser of two evils” problem.

A variation of IRV allows voters to rank groups of candidates equally. For example, a voter could rank candidates B and C equally for first choice, and D for second choice. For technical reasons that will not be discussed here, this equal-ranking option significantly mitigates the serious problems of IRV, but not enough to make it a good election method.

IRV does have one potential advantage over plurality. It requires the same voting equipment and the same voting mechanics (ranking candidates) as Condorcet voting. IRV could therefore possibly be transitional to Condorcet voting, a far superior alternative to IRV. However, IRV could also get entrenched and preclude Condorcet voting.

## Shortcomings of Instant Runoff

Instant Runoff has serious technical problems, since it fails the monotonic criteria:

*With the relative order or rating of the other candidates unchanged, voting a candidate higher should never cause the candidate to lose, nor should voting a candidate lower ever cause the candidate to win.*

This means that there are peculiar cases where the Instant Runoff method lets the “wrong” candidate win, even if the votes are all sincere. One example of such a case is the following votes count with four candidates “A, B, C, D”:

7: A, B, C  
6: B, A, C  
5: C, B, A

3 : D, C, B

Applying the rules of IRV, candidate A wins. But suppose the three voters who voted (D, C, B) now promote A from last choice all the way up to first choice, without changing the relative order of the other candidates. Now B wins instead of A. So by promoting A from last to first choice, those voters caused A to lose instead of win!

This is not the only inconsistency of the Instant Runoff voting method; other examples are discussed in the technical evaluation in **Appendix A**.

## *Condorcet voting*

The Condorcet method is the most sophisticated voting method. A complete discussion of the algorithm is reported in appendix A. Here, we will just give an informal description and a few examples. Condorcet method is a pairwise election method where each candidate runs against all the others separately, and one counts the “defeats” X/Y, i.e. the number of times candidates X beats candidates Y versus the number of times X is beaten by Y.

Defeats X/Y are ordered according their magnitude, the magnitude being the number of times candidates X beats candidates Y. For instance in this ballot with three candidates:

40 : C  
35 : A B C  
25 : B A C

We have the following defeats:

B/C:60/40 (B beats C 60 times, C beats B 40 times, the magnitude is 60)  
A/C:60/40 (A beats C 60 times, C beats A 40 times, the magnitude is 60)  
A/B:35/25 (A beats B 35 times, B beats A 25 times, the magnitude is 35)

Since A beats all its opponents in separate races, it is the clear winner. In more complex situations, no candidate beats all the others. Then the counting proceeds by removing the weakest defeats until an unbeaten candidate is found, or we get a tie. For instance in this case:

40 : C  
35 : A B C  
25 : B C

we get:

C/A: 65/35  
B/C: 60/40  
A/B: 35/25

and removing successively the weakest (smallest magnitude) defeat, we remain with the unbeaten candidate B which is the Condorcet winner. This is the algorithm describing the original Condorcet method, but actually PARTECS™ does not implement it (since it has some minor defects). PARTECS™ uses a state-of-the-art variation of Condorcet, the Cloneproof or Schwartz sequential dropping (SSD) Condorcet method, which is explained in **Appendix B**.

### Shortcomings of the Condorcet method

Condorcet is probably the soundest ordinal method available today and it is favoured by many academics. Nevertheless it has its own shortcomings users should be aware of.

- The Condorcet method is rather complex, difficult to explain and to grasp. Also, PARTECS™ does not implement its simplest version, Plain Condorcet, but implements the SSD Condorcet method, which is even more complex to explain;
- Condorcet results cannot easily be converted in percentages, so are difficult to read for people used to the more common voting methods;
- Condorcet is inferior to the Borda method in the case of sincere voters, since relevant information about the relative rating is lost (but this very fact increases the robustness against insincere votes);
- Still there are cases where Condorcet is not 100% robust against insincere voters. Below is an example of such a case.

### A case where Condorcet is not robust

Consider this (sincere) ballot:

40: C  
35: ABC  
25: BAC

Here the winner is A, who beats both B (35/25) and C (60/40). However, if the B voters strategically refuse to rank A while the A voters sincerely rank B, that will let B steal the election from A since

40: C  
35: ABC  
25: BC

would give the defeats:

C / A : 65/35  
B / C : 60/40  
A / B : 35/25

Removing the weakest defeat leaves B as the unbeaten candidate. Instant runoff and Borda method do not have this problem: voting C insincerely will go against the interest of B voters, since C will win, not B. However, it may be argued that these methods have even worse problems than Condorcet.

### What about ties?

In elections with a small number of voters (typical of decision making process in small committees), ties are relatively frequent and must be managed in some way. There are various ways to solve the ambiguities, for instance the President may decide, or a different voting method can be used to see if the ambiguity disappears. Here is an example with four candidates A, B, C, D and these votes by a six members committee:

3 : A C  
3 : B A

Nobody likes D, whereas three voters like A and three voters like B, but B voters also like A as a second choice, so we would expect A to win. However this is not the case using the Condorcet method, nor the Instant Runoff method. Let us consider first the Condorcet method tallying procedure. The defeats are:

A / D : 6/0  
A / C : 6/0  
C / D : 3/0  
B / D : 3/0

Dropping the weakest defeats we still have a tie between A and B since they are both unbeaten. Still, it is clear that A must be winner, since it gets both first and second choices. The ambiguity is removed by the Borda method:

A: 15  
B: 9  
C: 6  
D: 0

Condorcet tends to give more ties than other methods. This is a good thing in general, since it is better to return a tie in dubious cases, leaving the decision to a human, than to return a wrong result. Now, let's check if the Instant Runoff method gives a tie too:

```
This election required 2 rounds.  
Round #1:  
Removing D ...  
3: A C -  
3: B A -  
Round #2:  
Removing C ...  
.TIE EXCEPTION: No winner can be established, since the algorithm cannot  
decide the weakest candidate between A and B.
```

## What's the best voting method?

There is no best voting method.

You can always find a specific situation where a given voting method produces the wrong result. However, whereas in general there is no perfect voting method, in practice there are voting methods that can be worse or better for you and your organization.

Here are a few suggestions to guide you in the forest of voting methods. Take them with a grain of salt and judge for yourself, which is best in your specific situation.

### Questions and answers

**Q: I need to take a decision concerning a very small number of choices (2-3)**

A: consider plain Single Preference voting

**Q: I need to choose between a moderate number of choices and I want a simple and solid method...**

A: consider Approval method

**Q: I need to choose between a large number of choices; my voters have no reasons to be insincere...**

A: consider Borda method

**Q: My voters are familiar with Runoff method and would like to avoid voting twice...**

A: consider Instant Runoff method

**Q: I need to choose between a large number of choices; I want to be as robust as possible against insincere votes...**

**A: consider Condorcet method**

In most situations the simplest voting methods are the best solution: so consider plain Single Preference voting and Multiple Preference approval before any other method. Sometimes you may want to use more than a single voting method, and may want to study the robustness of a result against different voting methods. This is possible and made very easy by the PARTECS™ software.

In the case of small committees with highly skilled voters, one may consider using sophisticated methods. In this case Condorcet is usually a good choice. It may easily produce ties, however, so it may be a good idea to confront the result with the Borda method results, which are much less likely to end up in a tie.

## Appendix A: Technical Evaluation of Election Methods

*Taken with permission from [electionmethods.org](http://electionmethods.org) (Mike Ossipoff and Robert Paielli)*

An election method is a voting procedure and a set of mathematical rules for determining the winner(s). The best election method gives the electorate, to the maximum extent possible, the leaders they sincerely prefer, and it minimizes their need to vote strategically (e.g., for the “lesser of two evils”). The choice of an election method should not be based on subjective notions, nor should it be designed to advance any particular social, political or ideological agenda (other than fair elections, of course). We believe it should be based, rather, on objective technical criteria. The criteria we choose are listed below, followed by a compliance table. Of those criteria, we consider Monotonicity mandatory, and we consider compliance with Condorcet and other criteria highly desirable.

A few notes are in order regarding the assumptions and definitions upon which our criteria are based. We assume that each voter has a “sincere” ranking of the candidates according to the voter’s own criteria. A voter’s rank list can include one or more of the candidates. In other words, each voter has a first choice, and perhaps a second choice, a third choice, and so on. Ties or equal rankings are allowed. In reality, of course, many voters may be undecided or not know enough about some candidates to rank them, but we ignore such indecision and ignorance for our purposes here. If a particular voter ranks one candidate over another, the voter is said to “prefer” that candidate over the other. Other definitions are given below where applicable.

Depending on the election method, the voter’s “sincere” ranking of the candidates may not be the wisest way for the voter to actually vote. In plurality, for example, a “sincere” vote would be for the voter’s first choice, but that may not be wise if that first choice is not for one of the two major parties. Similarly, in IRV the voter will often be compelled to insincerely rank a “lesser of two evils” as his first choice. As we show below, voters have little incentive to reorder their sincere rankings in a Condorcet system. In Approval, the voter never needs to insincerely change the order of his sincere rankings, he only needs to decide where best to draw the line between approved and disapproved candidates.

- Monotonicity Criterion (MC)
- Condorcet Criterion (CC)
- Generalized Condorcet Criterion (GCC)
- Strategy-Free Criterion (SFC)
- Generalized Strategy-Free Criterion (GSFC)
- Strong Defensive Strategy Criterion (SDSC)

- Weak Defensive Strategy Criterion (WDSC)
- Favourite Betrayal Criterion (FBC)
- Participation Criterion (PC)
- Summability Criterion (SC)

Table 1: Election method criteria compliance (X = complies, - = does not comply)

Method	MC	CC	GCC	SFC	GSFC	SDSC	WDSC	FBC	PC	SC
Condorcet	X	X	X	X	X	X	X	-	-	X
Approval	X	-	-	-	-	-	X	X	X	X
Cardinal	X	-	-	-	-	-	X	X	X	X
Plurality	X	-	-	-	-	-	-	-	X	X
Borda	X	-	-	-	-	-	-	-	X	X
IRV	-	-	-	-	-	-	-	-	-	-

Note that the Condorcet method has several possible variations for resolving cyclical ambiguities, but Table 1 applies to the SSD (Schwartz Sequential Dropping), CSSD (Cloneproof SSD), and Beatpath Winner methods, which are explained elsewhere at this website.

The election methods considered here are far from a complete list of all methods that have ever been seriously proposed, but they are the methods we consider most important. We consider Condorcet the best choice, because it is the only method that complies with both the Monotonicity and Condorcet criteria, as well as several others. We consider Approval a good second choice, with the advantage of extreme simplicity. Unlike Condorcet, Approval Voting has a realistic chance of being adopted in the near term. Cardinal Ratings are strategically equivalent to Approval but more difficult to implement, hence they are perhaps not worth pursuing in the near term.

Plurality is important only because it is the current election method. We consider Instant Runoff Voting (IRV) the worst choice, but it is important because unfortunately it is currently popular among electoral reform organizations. Borda is not a serious contender but is included here because it is relatively well known.

## Monotonicity Criterion (MC)

### Statement of Criterion

*With the relative order or rating of the other candidates unchanged, voting a candidate higher should never cause the candidate to lose, nor should voting a candidate lower ever cause the candidate to win.*

## Complying Methods

All the methods listed in the compliance table above are monotonic except Instant Runoff Voting (IRV).

## Commentary

In the ordinal methods (Condorcet, Borda, and IRV), a candidate is “voted higher” by being ranked higher. In Approval Voting, a candidate is “voted higher” by being “approved” rather than “disapproved”. In a conventional plurality system, a candidate can be “voted higher” only by being voted for at all rather than not voted for.

Monotonicity is perhaps the most fundamental criterion for election methods. Common sense tells us that good election methods should be monotonic. Methods that fail to comply are erratic. A simple example will prove that IRV is non-monotonic. Consider, for example, the following vote count with three candidates “A, B, C”:

8 : A, C  
5 : B, A  
4 : C, B

In this example, eight voters ranked the candidates (A, C), five ranked them (B, A), and four ranked them (C, B). Candidate C was ranked first by the fewest voters and is eliminated. Since all the voters who ranked C first also ranked B second, B now has nine top-choice votes and wins. Suppose, however, that two of the voters who had ranked A first reverse their first two preferences so their votes change from (A, C) to (C, A). Now the vote count is:

6 : A, C  
5 : B, A  
4 : C, B  
2 : C, A

Candidate B is now ranked first by the fewest voters and is eliminated. Since the five voters who ranked B first also ranked A second, A now has eleven top-choice votes and wins. Hence, the two voters who demoted A from first to second choice caused A to win. That is, they caused A to win by ranking A lower, without changing the relative ordering of the other candidates. IRV therefore fails monotonicity. For an even more bizarre example, consider the following vote count with four candidates “A, B, C, D”:

7: A, B, C  
6: B, A, C  
5: C, B, A  
3: D, C, B

Applying the rules of IRV, candidate A wins. But suppose the three voters who voted (D, C, B) now promote A from last choice all the way up to first choice, without changing the relative order of the other candidates. Now B wins instead of A. So by promoting A from last to first choice, those voters caused A to lose instead of win. An election method that allows such nonsensical anomalies is erratic and should be rejected.

These are hardly contrived theoretical examples without practical relevance. IRV has serious problems both in theory and in practice. In practice, voters would soon realize, or be advised, that they cannot safely vote sincerely, and the political system would likely remain bogged down in a two-party duopoly just as it is today. And that is the optimistic scenario. If a third party somehow manages to become a strong contender, it could throw the entire political system into chaos, just as it could in our current plurality system.

## *Condorcet Criterion (CC)*

### Statement of Criterion

*If all votes are sincere, the Ideal Democratic Winner should win if one exists.*

### Definitions

A sincere vote is one with no falsified preferences or preferences left unspecified when the election method allows them to be specified (in addition to the preferences already specified). One candidate is preferred over another candidate if, in a one-on-one competition, more voters prefer the first candidate than prefer the other candidate. If one candidate is preferred over each of the other candidates, that candidate is the Ideal Democratic Winner (IDW).

### Complying Methods

The Condorcet method complies with the Condorcet Criterion, but none of the other methods in the compliance table above comply.

### Commentary

The Condorcet criterion is one of the most basic criteria for election methods. When an Ideal Democratic Winner exists, common sense tells us that ideally he or she should win. However,

the only method listed in Table 1 that complies with the Condorcet criterion is the Condorcet method itself, which is designed specifically to comply with the criterion named after it.

Non-ranking methods such as Plurality and Approval could not possibly comply with the Condorcet Criterion because they do not allow each voter to fully specify their preferences. But IRV allows each voter to rank the candidates, yet it still does not comply. A simple example will prove that IRV fails to comply with the Condorcet Criterion.

Consider, for example, the following vote count with three candidates "A, B, C":

8 : A, B  
7 : C, B  
5 : B

In this case, B is preferred to A by 12 votes to 8, and B is preferred to C by 13 to 7, hence B is preferred to both A and C. So according to common sense and the Condorcet criteria, B should win. But under IRV, B does not win. According to the rules of IRV, B is ranked first by the fewest voters and is eliminated. Again, an election method that allows such nonsensical anomalies should be rejected.

## ***Generalized Condorcet Criterion (GCC)***

### **Statement of Criterion**

*If all votes are sincere, the winner should be a member of the Smith set.*

### **Definitions**

A sincere vote is one with no falsified preferences or preferences left unspecified when the election method allows them to be specified (in addition to the preferences already specified). One candidate is preferred over another candidate if, in a one-on-one competition, more voters prefer the first candidate than prefer the other candidate.

The Smith set is the smallest set of candidates such that every member of the set is preferred to every candidate not in the set. If the Smith set consists of only one candidate, that candidate is the Ideal Democratic Winner (IDW).

## Complying Methods

The Condorcet method complies with the Generalized Condorcet Criterion, but none of the other methods in the compliance table above comply.

## Commentary

GCC generalizes the Condorcet Criterion (CC) to the case in which no Ideal Democratic Winner (IDW) exists, thereby covering all possible cases. If no IDW exists, then a cyclical ambiguity exists among the members of the Smith set, and that ambiguity must be resolved in such a way that the winner comes from that set. The commentary for CC above applies here also.

## *Strategy-Free Criterion (SFC)*

### Statement of Criterion

*If an Ideal Democratic Winner (IDW) exists, and if a majority prefers the IDW to another candidate, then the other candidate should not win if that majority votes sincerely and no other voter falsify any preferences.*

### Definitions

A sincere vote is one with no falsified preferences or preferences left unspecified when the election method allows them to be specified (in addition to the preferences already specified). One candidate is preferred over another candidate if, in a one-on-one competition, more voters prefer the first candidate than prefer the other candidate. If one candidate is preferred over each of the other candidates, that candidate is the Ideal Democratic Winner (IDW).

## Complying Methods

The Condorcet method complies with the Strategy-Free Criterion, but none of the other methods in the compliance table above comply.

## Commentary

The reader may be wondering how the IDW, if one exists, could possibly not be preferred by a majority of voters over any other candidate. The key is that some voters may have no preference between a given pair of candidates. Out of 100 voters, for example, 45 could prefer the IDW to another particular candidate, and 40 could prefer the opposite, with the other 15

having no preference between the two. In that case, it is not true that a majority of voters prefer the IDW to the other candidate, and SFC does not apply.

In order to understand SFC, one must also understand that there are two types of insincere votes: false preferences and truncated preferences. Voters truncate by terminating their rank list before their true preferences are fully specified (note that the last choice is always implied, so leaving it out is not considered truncation). Voters falsify their preferences, on the other hand, by reversing the order of their true preferences or by specifying a preference they don't really have. Suppose, for example, that a voter's true preferences are (A, B, C, D). The vote (A) or (A, B) would be a truncated vote, and the vote (B, A, C) or (A, C, B) would be a falsified vote.

SFC requires that the majority of voters who prefer the IDW to another particular candidate vote sincerely (neither falsify nor truncate their preferences), and it also requires that no other voter falsify preferences. SFC therefore implies that the minority that does not prefer the IDW to the other candidate cannot cause the other candidate to win by truncating their preferences. (In theory, that minority could cause the other candidate to win by falsifying their preferences, but that would be a very risky offensive strategy that is more likely to backfire than to succeed.) The significance of the SFC guarantee is that the majority has no need for defensive strategy, hence the name Strategy-Free Criterion.

The Condorcet election method was shown to comply with both the Condorcet and Generalized Condorcet Criteria (CC and GCC) above. Although compliance with CC and GCC are important, those criteria apply only in the theoretically ideal case in which all votes are sincere. The Strategy-Free criterion goes further and shows that, under certain reasonable conditions, a majority of voters have no incentive to vote insincerely. The fact that the Condorcet also complies with SFC therefore enhances the significance of CC and GCC considerably.

## *Generalized Strategy-Free Criterion (GSFC)*

### Statement of Criterion

*If a majority prefers a member of the Smith set to another candidate who is not in the Smith set, then the other candidate should not win if that majority votes sincerely and no other voter falsifies any preferences.*

## Definitions

A sincere vote is one with no falsified preferences or preferences left unspecified when the election method allows them to be specified (in addition to the preferences already specified). One candidate is preferred over another candidate if, in a one-on-one competition, more voters prefer the first candidate than prefer the other candidate. The Smith set is the smallest set of candidates such that every member of the set is preferred to every candidate not in the set. If the Smith set consists of only one candidate, that candidate is the Ideal Democratic Winner (IDW).

## Complying Methods

The Condorcet method complies with the Generalized Strategy-Free Criterion, but none of the other methods in the compliance table above comply.

## Commentary

GSFC generalizes the Strategy-Free Criterion (SFC) to the case in which no Ideal Democratic winner (IDW) exists, thereby covering all possible cases. If no IDW exists, then a cyclical ambiguity exists among the members of the Smith set and must be resolved. The commentary for SFC above applies here also.

## *Strong Defensive Strategy Criterion (SDSC)*

### Statement of Criterion

*If a majority prefers one particular candidate to another, then they should have a way of voting that will ensure that the other cannot win, without any member of that majority reversing a preference for one candidate over another or falsely voting two candidates equal.*

## Definitions

A voter votes X equal to Y if the voter doesn't vote X over Y, and doesn't vote Y over X, but votes X over someone, and votes Y over someone. A sincere vote is one with no falsified preferences or preferences left unspecified when the election method allows them to be specified (in addition to the preferences already specified). One candidate is preferred over another candidate if, in a one-on-one competition, more voters prefer the first candidate than prefer the other candidate.

## Complying Methods

The Condorcet method complies with the Strong Defensive Strategy Criterion, but none of the other methods in the compliance table above comply.

## Commentary

Compliance with SDSC means that a majority never needs any more than truncation strategy to defeat a particular candidate, even when countering offensive order reversal by that candidate's voters. Offensive order reversal is the only strategy that can create the need for defensive strategy in a Condorcet voting system.

## *Weak Defensive Strategy Criterion (WDSC)*

### Statement of Criterion

*If a majority prefers one particular candidate to another, then they should have a way of voting that will ensure that the other cannot win, without any member of that majority reversing a preference for one candidate over another.*

## Complying Methods

The Condorcet and Approval methods comply with the Weak Defensive Strategy Criterion, but none of the other methods in the compliance table above comply.

## Commentary

WDSC is identical to the Strong Defensive Strategy Criterion (SDSC), except that the phrase “or falsely voting two candidates equal” is removed from the end. That difference allows the Approval method to comply.

## *Favourite Betrayal Criterion (FBC)*

### Statement of Criterion

*For any voter who has a unique favourite, there should be no possible set of votes cast by the other voters such that the voter can optimize the outcome (from his own perspective) only by voting someone over his favourite.*

## Definition

A voter optimizes the outcome (from his own perspective) if his vote causes the election of the best possible candidate that can be elected, based on his own preferences, given all the votes cast by other voters.

## Complying Methods

The Approval method complies with the Favourite Betrayal Criterion, but none of the other methods in the compliance table above comply.

## Commentary

Election methods that meet this criterion provide no incentive for voters to betray their favourite candidate by voting another candidate over him or her. Although Condorcet technically fails to comply with FBC, the probability is small that a voter can cause a preferable result by not voting for his or her favourite in a Condorcet system.

## *Participation Criterion (PC)*

### Statement of Criterion

*Adding one or more ballots that vote X over Y should never change the winner from X to Y.*

### Complying Methods

Plurality, Approval, Cardinal Ratings, and Borda all pass the Participation Criterion. Both Condorcet and IRV fail.

### Commentary

The failure to pass the Participation Criterion is a minor embarrassment for Condorcet. We include it on this page to show that we are not trying to hide any shortcomings of Condorcet, but we do not consider it as fundamental as Monotonicity. As for IRV, it doesn't pass any of our other criteria either, so its failure here is hardly a surprise.

## *Summability Criterion (SC)*

### Statement of Criterion

*Each vote should map onto a summable array, where the summation operation is associative and commutative, and the winner should be determined from the array sum for all votes cast.*

### Complying Methods

All of the methods in the compliance table above comply with the summability criterion except Instant Runoff Voting (IRV).

### Commentary

The summability criterion is the only criteria discussed on this webpage that addresses implementation logistics. Election methods that comply with the summability criterion are substantially easier to implement with integrity than those that do not. All the election methods listed in Table 1 comply except Instant Runoff Voting (IRV).

In plurality voting, each vote is equivalent to a one-dimensional array with a 1 in the element for the selected candidate, and a 0 for each of the other candidates. The sum of the arrays for all the votes cast is simply a list of vote counts for each candidate.

Approval voting is the same as plurality voting except that more than one candidate can get a 1 in the array for each vote. Each of the selected or "approved" candidates gets a 1, and the others get a 0.

In Condorcet voting, each vote is equivalent to a two-dimensional array referred to as a pairwise matrix. If candidate A is ranked above candidate B, then the element in the A row and B column gets a 1, while the element in the B row and A column gets a 0. The pairwise matrices for all the votes are summed, and the winner is determined from the resulting pairwise matrix sum.

IRV does not comply with the summability criterion. In the IRV system, a count can be maintained of identical votes, but votes do not correspond to a summable array. The total possible number of unique votes grows factorially with the number of candidates. The larger the number of candidates, the more error-prone and less practical it becomes to maintain counts of each possible unique vote. It becomes impractical with more than about six candidates.

Suppose, for example, that the number of candidates is ten. In our current plurality system, the votes at any level (precinct, county, state, or national) can be compressed into a list of ten numbers. The same is true for an Approval system. For a Condorcet system, a 10x10 matrix is needed. In an IRV system, however, the number of possible unique votes is over ten factorial - a huge number.

Under IRV, therefore, every individual vote (rank list) must be available at a central location to determine the winner. In a major public election, that could be millions or even tens of millions of votes. The votes cannot be compressed by summing as in other election methods because votes may need to be transferred according to which candidates are eliminated in each round.

IRV therefore requires far more data transfer and storage than the other methods. Modern networking and computer technology can handle it, but that is beside the point. The biggest challenge in using computers for public elections will always be security and integrity. If many thousands of times more data needs to be transferred and stored, verification becomes more difficult and the potential for fraudulent tampering becomes substantially greater.

To illustrate this point, consider the verification of a vote tally for a national office. In our current plurality system, each precinct verifies its vote count. The counts for each precinct in a county can then be added to determine the county totals, and anyone with a calculator or computer can verify that the totals are correct. The same process is then repeated at the state level and the national level.

The point is that once the votes are verified at the lowest (precinct) level, the numbers are available to anyone for independent verification, and election officials could never get away with “fudging” the numbers. At the lowest level, ballot problems such as “hanging chads” could be a problem, but adding the vote counts will certainly not be a problem. And this applies not only to conventional plurality elections; it applies also to Condorcet, Approval, and even Borda - but not IRV.

In an IRV election, the voting data cannot be “compressed” by adding the vote totals together at each level, so verification of the tally results becomes nearly impossible. The final result depends on all the votes, but even if the individual votes are all counted correctly, nobody can verify that the total pool of votes has not been tampered with at some level of the tallying process. And with IRV's erratic properties, someone could lower the rankings of a candidate to make him win or raise the rankings of a candidate to make him lose. It's a prescription for disaster and voter cynicism.

If IRV were superior otherwise, then its failure to comply with the summability criterion might be excusable. But IRV has been shown to fail with respect to every one of the criteria listed, including such basic criteria as monotonicity. To accept the additional security risks that IRV poses would therefore be the epitome of folly.

## Appendix B: Formal discussion of the Condorcet method

Taken with permission from [electionmethods.org](http://electionmethods.org) (Mike Ossipoff and Robert Paielli)

In the Condorcet election method, voters rank the candidates in order of preference. The vote counting procedure then takes into account each preference of each voter for one candidate over another. It does so by conceptually breaking the election down into a series of separate races between each possible pairing of candidates, hence it is sometimes referred to as a “pairwise” method. If one of the candidates beats each of the other candidates in their one-on-one race, then that candidate wins. Otherwise, the result is ambiguous and a standard procedure is used to resolve the ambiguity. Unlike conventional plurality voting, Condorcet voting gives voters little incentive to falsify their true preferences.

### *Pairwise Matrices*

The basics of Condorcet voting are best illustrated by example. Suppose an election has four candidates designated A, B, C, and D. Each voter ranks the candidates in order of preference. For example, the vote (B, D, C) ranks B first, D second, and C third. The last choice is implicit. Voters are not required to fully rank the entire list. For example, the vote (D, B) indicates that the voter has no preference between A and C. Each vote translates to a pairwise matrix showing the equivalent vote for each possible pairing of candidates, as illustrated in the tables below.

Table 1: pairwise matrix for vote (B, D, C)

-	A	B	C	D
A	-	0	0	0
B	1	-	1	1
C	1	0	-	0
D	1	0	1	-

Table 2: pairwise matrix for vote (D, B)

-	A	B	C	D
A	-	0	0	0
B	1	-	1	0
C	0	0	-	0
D	1	1	1	-

Table 3: pairwise matrix sum for previous votes

-	A	B	C	D
A	-	0	0	0
B	2	-	2	1
C	1	0	-	0
D	2	1	2	-

Table 1 shows the pairwise matrix for the vote (B, D, C). Since B is ranked first, the B row gets a 1 in each of the columns for the other three candidates. Since D is ranked above C and A, the D row gets a 1 in the C and A columns. Finally, since C is ranked above A, the C row gets a 1 in the A column. Since A is not ranked, the A row is filled with zeros. All cells that do not get a 1 get a 0 (the diagonal cells are always empty). As another example, Table 2 shows the pairwise matrix for the vote (D, B). The pairwise matrix for each vote is summed to get the pairwise matrix sum. Table 3 shows the sum for the two votes discussed here. Corresponding elements of the individual pairwise matrices are simply added together to get the pairwise matrix sum.

The final pairwise matrix sum is used to determine the winner. If one of the candidates wins his or her one-on-one race with each of the other candidates, that candidate wins. For example, consider the pairwise matrix sum shown in Table 4 below. In this example, B beats A by 87-63, B beats C by 78-72, and B beats D by 73-51. Because B beats each of the other candidates, B wins.

Table 4: simple pairwise matrix

-	A	B	C	D
A	-	63	89	57
B	87	-	78	73
C	69	72	-	74
D	67	51	52	-

Table 5: ambiguous pairwise matrix

-	A	B	C	D
A	-	40	22	13
B	37	-	50	50
C	30	35	-	25
D	20	60	20	-

## Cyclic Ambiguity Resolution

Sometimes no candidate beats each of the other candidates. The result is then ambiguous, and the ambiguity must be resolved. Such cyclic ambiguities are true ambiguities in the preferences of the electorate, and the fact that Condorcet accurately reflects them is not a problem with the Condorcet method itself, as is often erroneously assumed. Several excellent methods exist

for resolving ambiguities, each with minor advantages and disadvantages compared to the other methods. The determination of the preferable method is an ongoing area of research and discussion. Two of those methods will be discussed here.

The ambiguous pairwise matrix of Table 5 will be used as an example to show how cyclical ambiguities are resolved. In this example, none of the candidates is unbeaten: A is beaten by D and C; B is beaten by A and D; C is beaten by B; and D is beaten by C. The notation A/B (“A over B”) will be used to denote the defeat of B by A. A cyclic ambiguity is a set of defeats that “wraps around” on it. For example, the list of defeats A/B, B/C, C/A forms a cycle. A cycle always involves at least three and possibly more candidates. A cycle also involves at least three and possibly more defeats.

The magnitude of a defeat is the number of votes cast against the defeated candidate. For example, in Table 5, D defeated B and got 60 votes against B in doing so; hence the magnitude of the defeat is 60 votes. Contrary to intuition, the margin of victory or defeat is irrelevant because dropping or ignoring a defeat, when necessary, is tantamount to overruling those votes for the defeat - and fairness requires that as few votes be overruled as possible. Note that the 50 votes against the defeat of B by D were “overruled” by the voters themselves. Some versions of Condorcet ambiguity resolution are based on margins, but those methods suffer from strategy problems that the methods explained here do not. See “Condorcet Rules: Winning Votes or Margins?” for more information on this topic.

The notation D/B (60) will be used to denote the defeat of B by D with a magnitude of 60 votes. Here are the defeats in Table 5, ordered by magnitude:

1. D/B (60)
2. B/C (50)
3. A/B (40)
4. C/A (30)
5. C/D (25)
6. D/A (20)

Two methods of cyclic ambiguity resolution will be discussed: Basic Condorcet and Schwartz Sequential Dropping (SSD). Basic Condorcet is simpler, but SSD is the state of the art. Software that implements SSD is provided through a link below. If the number of candidates is three or less, Basic Condorcet and SSD give the same result, but for more than three candidates they can give different results.

## Basic Condorcet

The Basic Condorcet method of ambiguity resolution is the original method proposed by Condorcet himself. It can be stated as follows: drop the weakest (smallest magnitude) defeat, repeating if necessary until one of the candidates is unbeaten.

The Basic Condorcet method works as follows for the example shown above. D/A is the weakest defeat, so it is dropped. Candidate A still has another defeat, so no candidate is unbeaten yet. Now C/D is the weakest defeat, so it is dropped. Candidate D is now unbeaten and wins.

Although it is superior to plurality and Instant Runoff Voting, Basic Condorcet suffers from some technical deficiencies compared to the SSD method to be discussed below and is not recommended for use in actual public elections - unless its simplicity makes the difference between public acceptance or lack thereof.

## Schwartz Sequential Dropping (SSD)

The Schwartz Sequential Dropping (SSD) method has a “plain” version and the “cloneproof” version.

The cloneproof version gives no group or party any advantage or disadvantage for having additional candidates that are essentially “clones” of each other. Except for the case of ties, the two versions give the same result. When the number of voters is small, ties are likely and the cloneproof version is needed.

The cloneproof version is slightly more complicated than the plain version, but it works well regardless of the number of voters, so it will serve as our standard. The cloneproof SSD procedure can be stated as follows:

Determine the Schwartz set. The Schwartz set is the innermost unbeaten set, or the smallest set of candidates such that any candidate outside the set beats none. If no defeats exist among the Schwartz set, then its members are the winners (plural only in the case of a tie, which must be resolved by another method). Otherwise, drop the weakest defeat among the Schwartz set, determine the new Schwartz set, and repeat the procedure.

The ordered list of defeats for the example is repeated here for convenience:

1. D/B (60)
2. B/C (50)
3. A/B (40)
4. C/A (30)
5. C/D (25)
6. D/A (20)

The SSD method works as follows for this example. Initially the entire set of candidates constitutes the Schwartz set because it doesn't contain a smaller unbeaten set. The weakest defeat is D/A, so it is dropped. The innermost unbeaten set is now still the set of all the candidates, and the smallest defeat is now C/D, so it is dropped. Candidate D is now unbeaten and wins. Note that Basic Condorcet and SSD produced the same winner in this case.

The cloneproof SSD method of resolving cyclic ambiguities has been found to be equivalent to another method called Beatpath Winner, which can be stated as follows:

- X has a beatpath to Y if X beats Y or if X beats another candidate that has a beatpath to Y.
- A sequence of defeats that makes it possible to say that X has a beatpath to Y is called a beatpath from X to Y.
- The strength of a beatpath is measured by the strength of its weakest defeat.
- X has a beatpath win against Y if the strongest beatpath from X to Y is stronger than the strongest beatpath from Y to X.
- The winner (or winners in the case of a tie) is the candidate against whom no candidate has a beatpath win.

## *Condorcet Rules: Winning Votes or Margins?*

When every candidate has a pairwise defeat by another candidate, an ambiguity exists in the will of the electorate. Condorcet's method, in all its versions, resolves the ambiguity by sequentially dropping the weaker defeats or by sequentially keeping the stronger defeats, which basically amounts to the same thing.

The question therefore arises as to how to measure the strength of a defeat. If Smith beats Jones by 60 votes to 50, what is the strength of Jones' defeat? One school of thought says that the strength of the defeat is the margin of defeat, or  $60-50=10$  votes in this case. That seems reasonable enough, but another school argues that the better measure of the strength of Jones' defeat in this case is 60 votes, the number of votes cast against Jones. We refer to the former measure as "margins" and the latter as "winning votes" (wv). For reasons to be explained, we strongly recommend the wv measure.

When every candidate has a pairwise defeat, and we have to elect one anyway, then that means that we have to ignore, disregard, or overrule someone's pairwise defeat(s) - when we

elect someone in spite of his having a pairwise defeat, a public statement that the voters prefer someone else to him. So Condorcet's practice of dropping weak defeats is natural and necessary.

But dropping or ignoring a defeat is not something to be taken lightly. It means that we're disregarding, or overruling, a statement made by the voting public, when they indicated that they preferred one candidate to another. And when we overrule that public choice, we're overruling those voters who won that particular public decision.

Suppose that, in the pairwise comparison between D and B, D beats B by 60 to 50, meaning that 60 people ranked D over B, and 50 people ranked B over D. If we drop that defeat, overrule that public statement that D is better than B, then we're also overruling the 60 voters who won that public vote about that two-way contest between D and B.

We want to minimize the number of voters we overrule. So we measure the importance of a defeat by the number of people who voted for that defeat.

Now, someone might reply that if you keep that defeat, we're overruling the 50 voters who voted against it, the 50 voters who ranked B over D. No, we're not. Those 50 voters were overruled by the public vote in which the voters collectively said that they prefer D to B. The only way that the voting system overrules a public decision is when it drops a defeat, when it overrules a public decision for one candidate over another. We're not doing that when we keep the defeat in which D beat B. We are doing that if we drop that defeat, overruling that public decision for D over B.

That's why we believe the winning-votes (wv) measure is more democratic than margins, more ethical. But there's another reason why we prefer wv to margins:

When a margin of defeat is calculated by subtraction, majority information is destroyed. It then becomes impossible for the method to honour majority rule, because the method has lost the information about what majorities have said.

For example, for the defeat of B by D, we get the margin of defeat by subtracting 50 from 60 to get 10. We've erased the fact that a 60% majority have ranked D over B, indicating preference for D over B. If majority rule matters, therefore, and we believe it does, then we want wv instead of margins.

It's because of that majority rule advantage that wv Condorcet meets the criteria called SFC, GSFC, WDSC, and SDSC, which we define elsewhere at this site. Condorcet based on margins doesn't meet any of those criteria.

So if the goal is to protect majority rule, or to get rid of the lesser-of-two-evils problem, or to minimize the need for defensive strategy (that's the goal that SFC, GSFC, WDSC, and SDSC measure for), then we want wv instead of margins.

### **Nash Equilibrium**

A Nash equilibrium is an game outcome in which no one player can improve the result for himself by changing his play, if no one else changes their play. A "player" is taken here to mean a set of voters who all vote identically.

With Condorcet that measures defeats by margins, as with IRV, there are often situations (configurations of sincere voter preferences) in which the only Nash equilibrium, are ones in which some voters vote someone over their favourite in order to protect majority rule or protect the win of a Condorcet candidate (a candidate who, when compared separately to each one of the others, is preferred to him/her by more people than vice-versa).

With Approval or wv Condorcet, every situation has at least one Nash equilibrium in which no one reverses a sincere preference.

### ***Condorcet Rules: Six Variations***

In all Condorcet variations or versions, the race for each office is broken down into a set of one-on-one races between each possible pairing of the candidates. If one candidate beats each of the other candidates, that candidate wins, of course. However, if no candidate is unbeaten, then a cyclic ambiguity exists, and it must be resolved. Several variations exist on how this is done.

A cycle is a series of one-on-one defeats that "wraps around" on it. For example, the set "A beats B, B beats C, and C beats A" forms a cycle. A cycle consists of at least three defeats and involves at least three candidates.

Also, as explained in the previous article in this series, we measure the strength of the defeat in a one-on-one race as the number of votes for the winner of that particular race. Note that this is not the margin of victory - the number of votes for the loser is irrelevant.

### **Basic Condorcet (PC)**

Basic (or Plain) Condorcet is the simplest variation of Condorcet voting. Its rules can be stated as follows:

Any candidate who isn't beaten by another candidate wins. If no candidate is unbeaten, then drop the weakest defeat. Repeat until one candidate is unbeaten.

Basic Condorcet is not as technically advanced as some of the other Condorcet variations to follow, but it is a good method, and its simplicity could facilitate public acceptance.

### **Sequential Dropping (SD)**

Sequential Dropping (SD) is the next simplest variation of Condorcet voting. Its rules can be stated as follows:

Any candidate who isn't beaten by another candidate wins. If no candidate is unbeaten, then drop the weakest defeat that's in a cycle.

### **Ranked-Pairs (RP)**

The rules for Ranked-Pairs Condorcet voting can be stated as follows:

In order of stronger defeats first, consider each defeat in turn as follows: keep it if it doesn't conflict with already-kept defeats by forming a cycle with any of them (i.e., isn't in a cycle consisting only of itself and some already-kept defeats). When all defeats have been so considered, a candidate wins if he is left with no defeats.

Barring the possibility of a tie, there will be only one winner.

Let's consider an RP count in more detail. We start by considering the strongest defeat. Is it in a cycle with already-kept defeats? No, it can't be, because no other defeats have even been considered yet. Therefore the strongest defeat will always be kept.

Next, consider the second strongest defeat. Is it in a cycle consisting only of it and some already-kept defeats? No, because we've only considered one other defeat so far (the strongest one), and a cycle requires 3 defeats. Therefore, the second strongest defeat always gets kept too.

Next, we consider the third strongest defeat. Does it form a cycle with the two strongest defeats? Maybe. If so, this third strongest defeat is in a cycle with already kept defeats, so we drop it.

Next we consider the fourth strongest defeat. As before, we ask if it's in a cycle consisting only of it and any already-kept defeats. As the rules specify, we keep it if it doesn't cycle with already-kept defeats, otherwise we drop it.

The procedure is repeated until all defeats have been considered. The candidate with no kept defeats wins. Barring pairwise ties, there will only be one such candidate.

Now, what happens if there are two or more defeats that are equal in strength? Call those "tied defeats". Keep any tied defeat that isn't in a cycle consisting only of it and some already-kept defeats that are not among those tied defeats.

(Midcount ties, occurring when there are equal defeats, are a problem peculiar to Ranked-Pairs. A number of solutions for such ties have been proposed. The solution described above is the simplest. It's perfectly adequate, because, in public elections, these ties, like any kind of ties, are extremely unlikely).

Ranked Pairs (RP), is a high-quality Condorcet variation and is popular with Condorcet voting experts. Note that Ranked Pairs is not the most descriptive name, but it's currently the most popular name. Niklaus Tideman coined it when he published that interpretation of Condorcet's "keep-strongest-defeats" proposal. A better descriptive name is Maximize Affirmed Majorities (MAM), coined by Steve Eppley, an expert on that method and its special properties.

Some people write definitions of RP that avoid mentioning cycles, speaking instead of defeats that are inconsistent or incompatible. But people will wonder what would make defeats incompatible, and it would still be necessary to define cycles. That's probably a drawback, because many people don't like mention of cycles.

## Schwartz Sequential Dropping (SSD)

We start with a definition of the Schwartz set:

1. An unbeaten set is a set of candidates of whom none is beaten by anyone outside that set.
2. An innermost unbeaten set is an unbeaten set that doesn't contain a smaller unbeaten set.
3. The Schwartz set is the set of candidates who are in innermost unbeaten sets.

Now, the rules for SSD:

1. If there's a candidate who isn't beaten by any other candidate, then that candidate wins.
2. Otherwise, calculate the Schwartz set, based only on undropped defeats.
3. Drop the weakest defeat among the candidates of that set. Go to 1.

SSD is another favourite Condorcet variation among experts.

## Cloneproof Schwartz Sequential Dropping (CSSD)

In public elections where the number of voters is large, the chance of pairwise ties is small. In the absence of pairwise ties, SSD and CSSD give the same result. However, SSD can have problems when the number of voters is small, as in small committees. Then CSSD is the better choice, although it is slightly more complicated. CSSD can also be used in public elections, of course.

CSSD is the same as SSD, except for the stopping rule. Whereas SSD stops the count when a candidate is unbeaten, CSSD stops the count only when there are no defeats among the candidates of the current Schwartz set.

Here are the CSSD rules:

1. Calculate the Schwartz set based only on undropped defeats.
2. If there are no defeats among the members of that set then they (plural in the case of a tie) win and the count ends.
3. Otherwise, drop the weakest defeat among the candidates of that set. Go to 1.

## Beatpath Winner

The brief Beatpath Winner algorithm most easily implements CSSD. Beatpath Winner is equivalent to CSSD in the sense that they always find the same winner(s). Beatpath Winner can be implemented in an elegantly brief algorithm. If that is an important consideration, then Beatpath Winner should be considered. Otherwise SSD or RP will probably be a more acceptable public proposal. A beatpath Winner proposal could be justified via the CSSD definition.

Here are the definitions and rules for the Beatpath Winner algorithm:

- X has a beatpath to Y if X beats Y or if X beats another candidate that has a beatpath to Y.
- A sequence of defeats that makes it possible to say that X has a beatpath to Y is called a beatpath from X to Y.
- The strength of a beatpath is measured by the strength of its weakest defeat.
- X has a beatpath win against Y if the strongest beatpath from X to Y is stronger than the strongest beatpath from Y to X.
- The winner (or winners in the case of a tie) is the candidate against whom no candidate has a beatpath wins.

Again, barring pairwise ties or equal defeats, there will be only one winner. We've defined six Condorcet variations: BC, SD, RP, SSD, CSSD, and Beatpath Winner. Except for Basic Condorcet, these variations have no significant differences in merit. The choice should be based entirely on which one would be more acceptable to the public or the enacting legislature.

## References

### General references on voting methods

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- The Debian Voting System, <http://seehuhn.de/comp/vote.html>

### Free Internet voting systems

- Condorcet Internet Voting Service <http://www.cs.cornell.edu/andru/civs.html>
- Online Condorcet Voting Computation, <http://www.ericgorr.net/condorcet>
- Caltech-MIT Voting Technology Project, <http://www.vote.caltech.edu>

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